Estimating the Value of Information

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Motivation

- How much would investors pay to receive investment-relevant information?
- Understanding the private incentives to collect information is a central issue for market efficiency
- Quantifying the value of information is key for:
  - pricing/ranking different information services
  - compensating macro and firm-level analysts
  - penalizing insider trading
  - improving information services for investors

This paper

- We present a framework for evaluating informative (but noisy) signals from the point of view of a utility maximizing investor
- Illustrate our framework by estimating the values of key macroeconomic indicators
- Provide comparative statics for the determinants of the value of information

Main idea

- Risk averse investor optimizes her portfolio and consumption using either
  1. prior probabilities on the states of nature, or
  2. posterior probabilities based on an information source (e.g., GDP report)
- Value of information is the price that renders her indifferent between the two cases
  - similar to Grossman and Stiglitz (1980) but more realistic preferences and markets
- Key ingredients: preferences, asset prices, prior/posterior probabilities (forward looking)

Prior and posterior probabilities

- We estimate prior and posterior probabilities from S&P 500 option prices around informational releases (say GDP growth)
  - Prior = probability distribution observed just before the signal is released
  - Posterior = probability distribution immediately after the signal is released
- Use this posterior to generate a "what if" analysis – allow the investor to trade using an updated distribution
- With a large sample of realized distribution changes we can estimate an average value of information

Why not use announcement returns?

- Price changes provide an indication of signal informativeness
  - Fama, Fisher, Jensen, and Roll (1969)
- But do not directly provide its economic value
- One needs a model of
  - preferences $\rightarrow$ willingness to trade on new information
  - investment opportunities $\rightarrow$ how can they trade
- Risk aversion and the willingness to substitute current and future consumption are particularly important

Preview of what we obtain

- We derive an estimable expression for the value of information associated with an information source
  - GMM estimation is natural
- Estimate values of information under standard preference parameters (discount rate, risk aversion, and EIS)
- Show how these change with preference parameters

Related literature - Psychic vs. instrumental value

- Cabrales et al. (2013 AER) study log utility agent faced with a static investment problem
  - Value of information equals mean reduction in entropy
  - We generalize to a dynamic environment and provide an estimation method
  - Log utility case is upper bound on "ruin-averse" preferences, but not on recursive utility, which we study
- Recursive utility agent may like early resolution of uncertainty
  - Entirely about the attitude of the agent toward uncertainty, even when she cannot alter her consumption plan
  - Epstein, Farhi, and Strzalecki (2014 AER) calibrate this psychic value of information
  - We estimate also the instrumental value of information reflecting the improvement in consumption and investment
  - Decompose the value of information into these two channels

Related literature - Private vs. public information

- We estimate the value of both:
  1. Private information: trade on information at stale prices
  2. Public information: trade at prices that reflect new information

  - Depart from literature focusing on public/social value (Hirshleifer, 1971 AER)
- Information acquisition / markets for information literature
  - Quantitative work in this field is rare, and has thus far relied on stronger assumptions
    - Savov (2014 JFE), Manela (2014 JFE)
  - We move beyond CARA utility to commonly used preferences
    - Can be important (Breon-Drish, 2015; Malamud, 2015)

State space and preferences

- Discrete time, infinite horizon
- Random state $z_t \in \{1, ..., n\}$
- Markovian state transition probabilities $p\left(z_{t+1}|z_t\right)$
- State prices $q\left(z_{t+1}|z_t\right) > 0$
  - no arbitrage

The agent's problem

- Price-taking consumer-investor with Epstein-Zin utility

$$V_t = \left[(1-\beta)\, c_t^{1-\rho} + \beta\mu\left[V_{t+1}\right]^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

V_t is utility starting at some date-t

- Certainty equivalent function is homogeneous

$$\mu\left[V_{t+1}\right] = \left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}$$

  - central role in ex-ante value of information
- Recursive preferences are widely used to fit asset pricing facts
  - $\rho = \gamma$ give expected utility with CRRA
  - $\rho = \gamma = 1$ give log utility

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Private information setup

- Agent can buy stream of signals s_t from information source α
  - GDP, Employment, ...
  - Matrix of conditional probabilities $\alpha\left(s_t|z_{t+1}\right)$
- Observing a signal, agent forms posterior probabilities $p_\alpha\left(z_{t+1}|s_t, z_t\right)$ and makes a consumption/investment decision

Order of Events During Time t			
State z_t realized	Investor observes signal s_t	Investor chooses consumption c_t and investment portfolio weights w_{t+1}	Signal s_t becomes public and prices adjust

- **Question**: How much would an agent be willing to pay to privately observe such a stream of signals?

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Public information setup

- Agent can buy stream of signals s_t from information source α
  - GDP, Employment, ...
  - Matrix of conditional probabilities $\alpha\left(s_t|z_{t+1}\right)$
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Order of Events During Time t			
State z_t realized	Investor observes signal s_t	Signal s_t becomes public and prices adjust	Investor chooses consumption c_t and investment portfolio weights w_{t+1}

- **Question**: How much would an agent be willing to pay to publicly observe such a stream of signals?

The value of information

- Merton-Samuelson consumption/investment problem albeit with an additional signal s:

$$V(a_t, z_t, s_t) = \max_{c_t, \mathbf{w}_{t+1}} \left\{ (1-\beta) c_t^{1-\rho} + \beta E_t \left[V(a_{t+1}, z_{t+1}, s_{t+1})^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right\}$$

s.t. the wealth constraint $a_{t+1} = (a_t - c_t) \sum_{i=1}^{n} w_{it+1} R_{it+1}$

Definition

The value of information structure α in state z_t is the fraction of wealth Ω the agent is willing to give up to observe a stream of signals s_t, s_{t+1}, ..., each generated by α

$$\mu[V(a_t(1-\Omega), z_t, s_t; \alpha) | z_t; \alpha] = V(a_t, z_t; \alpha_0)$$

where $\mu[\cdot]$ is the certainty equivalent over the signal s_t

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Convenient transformation

- Easier to work with the transformed value of information

$$\omega\left(z;\alpha\right)\equiv-\ln(1-\Omega\left(z;\alpha\right))$$

  - $\Omega\approx\omega$ when these are close to zero

- We can then write:

$$\omega\left(z_t;\alpha\right)=\log\mu\left[e^{\rho\{v(z_t,s_t;\alpha)-v(z_t;\alpha_0)\}}\middle|z_t;\alpha\right]\tag{1}$$

- Value of information depends on the (nonlinear) average improvement in the log value-to-consumption ratio $v\equiv\ln\frac{V}{c}$
  - $v\left(z_t,s_t;\alpha\right)$ is informed log value-to-consumption ratio
  - $v\left(z_t;\alpha_0\right)$ is uninformed log value-to-consumption ratio

Moment conditions for the value of information

- FOC for the agent's problem + some algebra yield:

$$E\left[e^{(\gamma-1)[\rho v(z_t;\alpha_0)+\omega(z_t;\alpha)]}\left\{1-\beta+\beta^{\frac{1}{\rho}}\Gamma\left(z_t,s_t;\alpha\right)^{\frac{\gamma(1-\rho)}{\rho(1-\gamma)}}\right\}^{\frac{\rho(1-\gamma)}{1-\rho}}-1|z_t\right]=0$$

with

$$\Gamma\left(z_t,s_t;\alpha\right)\equiv\sum_{z_{t+1}}\left\{q\left(z_{t+1}|z_t\right)^{\gamma-1}e^{(1-\gamma)\left[\rho v\left(z_{t+1};\alpha_0\right)+\omega\left(z_{t+1};\alpha\right)\right]}p_\alpha\left(z_{t+1}|z_t,s_t\right)\right\}^{\frac{1}{\gamma}}$$

- n moments with n unknown $\omega\left(z_t;\alpha\right)$ for $z_t\in\{1,...,n\}$
- Assumed "knowns": preference parameters β, γ, ρ, state prices q, posterior probabilities p, and the log value-to-consumption ratio without information $v\left(z_t;\alpha_0\right)$
- $\Gamma\left(z_t,s_t;\alpha\right)$ is the expectation of a non-linear function of (gross) asset returns $q\left(z_{t+1}|z_t\right)^{-1}$, future $v\left(z_{t+1};\alpha_0\right)$, and future values of information $\omega\left(z_{t+1};\alpha\right)$
- Agent values high payoffs in high value of information states

One-time signals

- Value of a one-time signal is sometimes more relevant

$$E\left[e^{(\gamma-1)[\rho v(z_t;\alpha_0)+\omega(z_t;\alpha)]}\left\{1-\beta+\beta^{\frac{1}{\rho}}\Gamma\left(z_t,s_t;\alpha\right)^{\frac{\gamma(1-\rho)}{\rho(1-\gamma)}}\right\}^{\frac{\rho(1-\gamma)}{1-\rho}}-1|z_t\right]=0$$

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Psychic vs. instrumental values of information

- We define the psychic value of information structure α as the fraction of wealth Ω^P the agent is willing to give up to obtain the same stream of signals considered above
  - But she is not allowed to change her consumption-investment plan relative to the uninformed α_0 benchmark case
  - Instead the only benefit from the signals comes from early resolution of uncertainty
- The instrumental value of information is the fraction of wealth Ω^I that an agent who acquired the stream of signals is willing to give up to be able to optimize her consumption-investment plan according to the signals
- Total value of information is approximately the sum of the psychic and instrumental values

$$\omega = \omega^P + \omega^I \tag{2}$$

Private vs. public information

- Value of public information is attained by a small tweak

$$E\left[e^{(\gamma-1)[\rho v(z_t;\alpha_0)+\omega(z_t;\alpha)]}\left\{1-\beta+\beta^{\frac{1}{\rho}}\Gamma\left(z_t,s_t;\alpha\right)^{\frac{\gamma(1-\rho)}{\rho(1-\gamma)}}\right\}^{\frac{\rho(1-\gamma)}{1-\rho}}-1|z_t\right]=0$$

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- The key difference between the private and public cases is that in the private case the agent can use the information before market prices react
- Psychic value of public information is identical to the private information case
- Instrumental value of public information can differ substantially from the private information counterpart
  - Intuitively, no instrumental value if price adjustments offset the potential gains from improved investment returns

Social vs. private value of information

- The psychic value is a pure gain in social welfare as opposed to a transfer among agents
- The instrumental value of private information constitutes a transfer from other investors in an exchange economy
  - A social value could arise from improved capital allocation to production Ai (2007 WP)

Estimation: main ideas

- FOC for the agent's problem + some algebra yield:

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- Take the parameters β, γ, and ρ as given
- Estimate discrete state prices from SPX options
- Estimate physical prior/posterior probabilities using parametric recovery (γ exponential tilting)
- Estimate the uninformed log-value-to-consumption ratios by solving a well-known fixed point problem
- Condition moments on information release dates to estimate their associated value of information, e.g. $\omega\left(z_t;\text{GDP}\right)$

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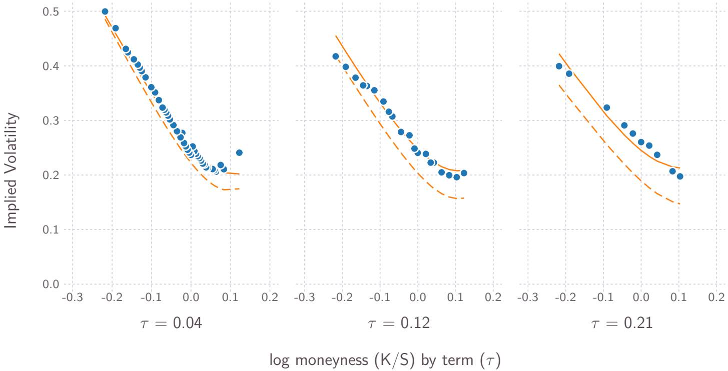
Implied volatility surface

December 4, 1998 Employment Report



Recovering the physical probability matrix

- Physical probabilities p_{ijt} are related to state prices q_{ijt} by a stochastic discount factor m_{ijt}

$$q_{ijt} = m_{ijt} p_{ijt} \tag{3}$$

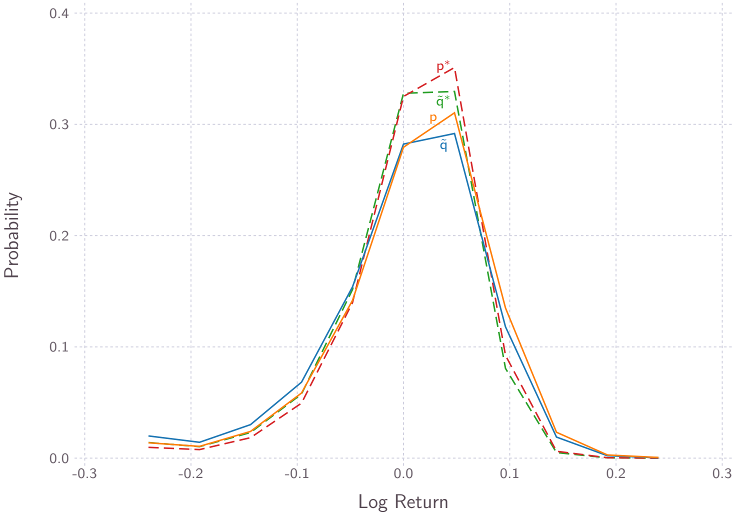
- We assume that physical probabilities at each time t are an exponentially-tilted version of state prices

$$p_{ijt} = \frac{e^{\epsilon r_p(z_{t+1}=j|z_t=i)} q_{ijt}}{\sum_k e^{\epsilon r_p(z_{t+1}=k|z_t=i)} q_{ikt}} \tag{4}$$

  - Securities paying in good states with high returns are relatively cheap, with the size of the wedge determined by risk aversion ϵ
  - Calibrate $\epsilon = 1.5$ to match equity premium over our sample
  - Commonly used in empirical options studies (e.g. Bakshi, Kapadia, and Madan, 2003; Bliss and Panigirtzoglou, 2004)

Prior and posterior risk-neutral and physical probabilities

December 4, 1998 Employment Report



Data

- Daily option prices from OptionMetrics, January 4, 1996 to August 31, 2015
- Commonly-used filters:
  - Restrict attention to at or out of the money calls and puts
  - At least seven days to maturity
  - Strictly positive volume
- Macroeconomic indicators release dates from Bloomberg's Economic Calendar

Informational events

Event	Source	Obs.
Consumer Comfort	Bloomberg	489
Employment	Bureau of Labor Statistics	194
FOMC Decision	Fed	131
GDP	Bureau of Economic Analysis	197
Jobless Claims	U.S. Department of Labor	804
Mortgage Applications	Mortgage Bankers' Association	481

Benchmark parameters

- Our benchmark parameters focus on parameters commonly used in the asset pricing literature:
  - time discount rate $\beta = 0.998$
  - relative risk aversion $\gamma = 10$
  - elasticity of intertemporal substitution $1/\rho = 1.5$
  - monthly horizon $\tau = 1/12$
- Bansal-Yaron (2004 JF) and subsequent literature calibrate these parameters to match key asset pricing moments such as the equity premium and volatility of the risk free rate

Estimated value of private information

Tbl 2: Willing to pay between 4 and 15 basis points of wealth for a one-time peek into the informational content of these leading macroeconomic indicators

Panel A: One-time Signal

	$RRA = 10,\ EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			
Event	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Obs
Consumer Comf.	0.039	(0.003)	0.039	0.034	(0.004)	0.033	0.417	(0.040)	0.413	574
Employment	0.054	(0.005)	0.053	0.061	(0.007)	0.060	0.565	(0.052)	0.554	207
FOMC	0.035	(0.005)	0.030	0.037	(0.007)	0.030	0.341	(0.045)	0.283	133
Pre-FOMC	0.038	(0.005)	0.037	0.032	(0.005)	0.032	0.406	(0.046)	0.395	134
GDP	0.034	(0.003)	0.034	0.033	(0.004)	0.033	0.364	(0.035)	0.360	206
Jobless Claims	0.043	(0.003)	0.043	0.041	(0.003)	0.040	0.445	(0.030)	0.441	887
Mortgage App.	0.035	(0.003)	0.034	0.034	(0.003)	0.032	0.357	(0.030)	0.346	570

Panel B: Signal Every Period

	$RRA = 10,\ EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			
Event	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Obs
Consumer Comf.	14.13	(1.22)	13.57	7.90	(0.76)	7.87	78.19	(0.12)	77.99	574
Employment	23.37	(2.01)	21.73	12.38	(1.12)	12.83	86.47	(0.02)	85.98	207
FOMC	14.64	(0.05)	12.12	7.42	(0.22)	6.15	69.76	(0.00)	62.57	133
Pre-FOMC	15.41	(0.66)	14.48	7.03	(1.22)	7.21	75.43	(0.03)	74.27	134
GDP	14.44	(0.95)	13.82	7.02	(0.66)	7.18	71.55	(0.01)	71.18	206
Jobless Claims	17.69	(1.17)	16.84	8.59	(0.53)	8.82	78.70	(0.05)	78.38	887
Mortgage App.	12.52	(0.93)	11.78	7.90	(0.76)	7.86	72.69	(0.10)	71.41	570

Estimated value of private information

Tbl 2: Willing to pay between 4 and 15 basis points of wealth for a one-time peek into the informational content of these leading macroeconomic indicators

Panel A: One-time Signal

	$RRA = 10,\ EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			
Event	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Obs
Consumer Comf.	0.039	(0.003)	0.039	0.034	(0.004)	0.033	0.417	(0.040)	0.413	574
Employment	0.054	(0.005)	0.053	0.061	(0.007)	0.060	0.565	(0.052)	0.554	207
FOMC	0.035	(0.005)	0.030	0.037	(0.007)	0.030	0.341	(0.045)	0.283	133
Pre-FOMC	0.038	(0.005)	0.037	0.032	(0.005)	0.032	0.406	(0.046)	0.395	134
GDP	0.034	(0.003)	0.034	0.033	(0.004)	0.033	0.364	(0.035)	0.360	206
Jobless Claims	0.043	(0.003)	0.043	0.041	(0.003)	0.040	0.445	(0.030)	0.441	887
Mortgage App.	0.035	(0.003)	0.034	0.034	(0.003)	0.032	0.357	(0.030)	0.346	570

Panel B: Signal Every Period

	$RRA = 10,\ EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			
Event	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Obs
Consumer Comf.	14.13	(1.22)	13.57	7.90	(0.76)	7.87	78.19	(0.12)	77.99	574
Employment	23.37	(2.01)	21.73	12.38	(1.12)	12.83	86.47	(0.02)	85.98	207
FOMC	14.64	(0.05)	12.12	7.42	(0.22)	6.15	69.76	(0.00)	62.57	133
Pre-FOMC	15.41	(0.66)	14.48	7.03	(1.22)	7.21	75.43	(0.03)	74.27	134
GDP	14.44	(0.95)	13.82	7.02	(0.66)	7.18	71.55	(0.01)	71.18	206
Jobless Claims	17.69	(1.17)	16.84	8.59	(0.53)	8.82	78.70	(0.05)	78.38	887
Mortgage App.	12.52	(0.93)	11.78	7.90	(0.76)	7.86	72.69	(0.10)	71.41	570

Psychic vs. instrumental value of private information

Tbl 3: Values of some one-time signals derives mostly from instrumental value as they improve agent's consumption-investment plan, while for others psychic value dominates

Panel A: One-time Signal

Event	$\Omega = 1 - e^{-\omega}$	$\omega = \omega^P + \omega^I$	ω^P	ω^I	$\lvert\omega^P\rvert/\lvert\omega\rvert$	$\lvert\omega^I\rvert/\lvert\omega\rvert$	Obs
All	0.043	0.043	0.000	0.043	0.000	100.000	4575
Consumer Comf.	0.039	0.039	0.000	0.039	0.014	99.986	574
Employment	0.054	0.054	-0.000	0.054	0.002	100.002	206
FOMC	0.035	0.035	0.000	0.035	0.006	99.994	130
Pre-FOMC	0.038	0.038	-0.000	0.038	0.004	100.004	131
GDP	0.034	0.034	0.000	0.034	0.014	99.986	202
Jobless Claims	0.043	0.043	0.000	0.043	0.021	99.979	875
Mortgage App.	0.035	0.035	-0.000	0.035	0.010	100.010	570

Panel B: Signal Every Period

Event	$\Omega = 1 - e^{-\omega}$	$\omega = \omega^P + \omega^I$	ω^P	ω^I	$\lvert\omega^P\rvert/\lvert\omega\rvert$	$\lvert\omega^I\rvert/\lvert\omega\rvert$	Obs
All	18.794	20.818	-0.005	20.823	0.024	100.024	4575
Consumer Comf.	14.129	15.233	-0.002	15.234	0.011	100.011	574
Employment	23.368	26.616	0.007	26.608	0.028	99.972	206
FOMC	14.636	15.824	0.005	15.819	0.034	99.966	130
Pre-FOMC	15.415	16.741	-0.006	16.747	0.034	100.034	131
GDP	14.435	15.590	0.002	15.588	0.012	99.988	202
Jobless Claims	17.689	19.467	-0.000	19.467	0.002	100.002	875
Mortgage App.	12.523	13.379	-0.004	13.383	0.032	100.032	570

Psychic vs. instrumental value of private information

Tbl 3: Values of some one-time signals derives mostly from instrumental value as they improve agent's consumption-investment plan, while for others psychic value dominates

Panel A: One-time Signal

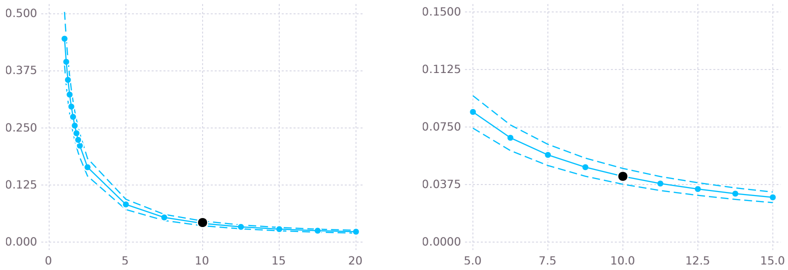
Event	$\Omega = 1 - e^{-\omega}$	$\omega = \omega^P + \omega^I$	ω^P	ω^I	$\lvert\omega^P\rvert/\lvert\omega\rvert$	$\lvert\omega^I\rvert/\lvert\omega\rvert$	Obs
All	0.043	0.043	0.000	0.043	0.000	100.000	4575
Consumer Comf.	0.039	0.039	0.000	0.039	0.014	99.986	574
Employment	0.054	0.054	-0.000	0.054	0.002	100.002	206
FOMC	0.035	0.035	0.000	0.035	0.006	99.994	130
Pre-FOMC	0.038	0.038	-0.000	0.038	0.004	100.004	131
GDP	0.034	0.034	0.000	0.034	0.014	99.986	202
Jobless Claims	0.043	0.043	0.000	0.043	0.021	99.979	875
Mortgage App.	0.035	0.035	-0.000	0.035	0.010	100.010	570

Panel B: Signal Every Period

Event	$\Omega = 1 - e^{-\omega}$	$\omega = \omega^P + \omega^I$	ω^P	ω^I	$\lvert\omega^P\rvert/\lvert\omega\rvert$	$\lvert\omega^I\rvert/\lvert\omega\rvert$	Obs
All	18.794	20.818	-0.005	20.823	0.024	100.024	4575
Consumer Comf.	14.129	15.233	-0.002	15.234	0.011	100.011	574
Employment	23.368	26.616	0.007	26.608	0.028	99.972	206
FOMC	14.636	15.824	0.005	15.819	0.034	99.966	130
Pre-FOMC	15.415	16.741	-0.006	16.747	0.034	100.034	131
GDP	14.435	15.590	0.002	15.588	0.012	99.988	202
Jobless Claims	17.689	19.467	-0.000	19.467	0.002	100.002	875
Mortgage App.	12.523	13.379	-0.004	13.383	0.032	100.032	570

Comparative statics I

Fig 4: One-time signal of jobless claims

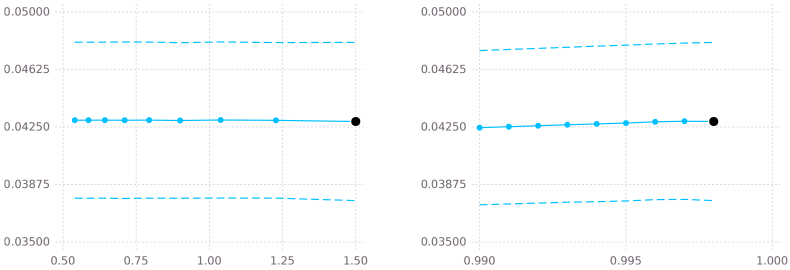


(a) Relative risk aversion $\gamma = \rho$ (b) Relative risk aversion γ

- Investment-relevant information is less useful to a more risk averse agent because her willingness to change her portfolio to take into account the information is limited
- Risk aversion effect weakens when EIS is held fixed
  - Counter effect of stronger preference for early resolution of uncertainty

Comparative statics II

Fig 4: One-time signal of jobless claims



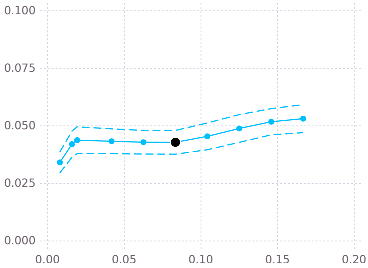
(a) Elasticity of intertemporal substitution $1/\rho$

(b) Time discount factor β

- Higher EIS makes agent more willing to use information to increase future consumption
- When the time discount factor β increases the value of information increases because the agent attaches more value to future periods

Comparative statics III

Fig 4: One-time signal of jobless claims



(a) Horizon in years τ

- By shrinking the horizon we better capture the value of information to a more active trader
- Shorter maturity options are less sensitive to the macro announcements, and therefore the value of information is mostly increasing on net in the investment horizon

Estimated value of *public* information

Tbl 4: As expected, values of public information are uniformly smaller than private values of information reported above

Panel A: One-time Signal

	$RRA = 10,\ EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			
Event	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Obs
Consumer Comf.	0.000	(0.000)	0.000	0.000	(0.000)	-0.001	0.010	(0.003)	0.007	574
Employment	0.002	(0.000)	0.001	0.003	(0.000)	0.002	0.032	(0.004)	0.021	207
FOMC	0.005	(0.000)	-0.000	0.007	(0.000)	-0.000	0.068	(0.006)	0.009	133
Pre-FOMC	-0.002	(0.000)	-0.003	-0.002	(0.000)	-0.002	-0.019	(0.004)	-0.030	134
GDP	0.001	(0.000)	0.000	0.000	(0.000)	-0.000	0.011	(0.001)	0.007	206
Jobless Claims	-0.000	(0.000)	-0.001	-0.001	(0.000)	-0.002	0.004	(0.004)	-0.000	887
Mortgage App.	-0.001	(0.000)	-0.002	0.000	(0.000)	-0.001	0.004	(0.004)	-0.008	570

Panel B: Signal Every Period

	$RRA = 10,\ EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			
Event	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Obs
Consumer Comf.	0.12	(0.06)	0.01	-0.02	(0.07)	-0.37	2.24	(0.66)	1.14	574
Employment	0.94	(0.10)	0.43	0.53	(0.04)	0.21	6.55	(0.52)	2.60	207
FOMC	1.92	(0.03)	-0.35	1.13	(0.02)	-0.45	12.66	(0.10)	-8.33	133
Pre-FOMC	-0.75	(0.05)	-1.24	-0.39	(0.01)	-0.45	-4.58	(0.38)	-8.49	134
GDP	0.23	(0.01)	0.08	-0.02	(0.01)	-0.10	1.02	(0.30)	-0.40	206
Jobless Claims	-0.28	(0.08)	-0.44	-0.44	(0.06)	-0.58	-1.09	(1.02)	-2.53	887
Mortgage App.	-0.28	(0.07)	-0.72	-0.00	(0.07)	-0.37	-1.17	(1.24)	-5.81	570

Estimated value of *public* information

Tbl 4: As expected, values of public information are uniformly smaller than private values of information reported above

Panel A: One-time Signal

	$RRA = 10,\ EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			
Event	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Obs
Consumer Comf.	0.000	(0.000)	0.000	0.000	(0.000)	-0.001	0.010	(0.003)	0.007	574
Employment	0.002	(0.000)	0.001	0.003	(0.000)	0.002	0.032	(0.004)	0.021	207
FOMC	0.005	(0.000)	-0.000	0.007	(0.000)	-0.000	0.068	(0.006)	0.009	133
Pre-FOMC	-0.002	(0.000)	-0.003	-0.002	(0.000)	-0.002	-0.019	(0.004)	-0.030	134
GDP	0.001	(0.000)	0.000	0.000	(0.000)	-0.000	0.011	(0.001)	0.007	206
Jobless Claims	-0.000	(0.000)	-0.001	-0.001	(0.000)	-0.002	0.004	(0.004)	-0.000	887
Mortgage App.	-0.001	(0.000)	-0.002	0.000	(0.000)	-0.001	0.004	(0.004)	-0.008	570

Panel B: Signal Every Period

	$RRA = 10,\ EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			
Event	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Ω	$se(\Omega)$	$\tilde{\Omega}$	Obs
Consumer Comf.	0.12	(0.06)	0.01	-0.02	(0.07)	-0.37	2.24	(0.66)	1.14	574
Employment	0.94	(0.10)	0.43	0.53	(0.04)	0.21	6.55	(0.52)	2.60	207
FOMC	1.92	(0.03)	-0.35	1.13	(0.02)	-0.45	12.66	(0.10)	-8.33	133
Pre-FOMC	-0.75	(0.05)	-1.24	-0.39	(0.01)	-0.45	-4.58	(0.38)	-8.49	134
GDP	0.23	(0.01)	0.08	-0.02	(0.01)	-0.10	1.02	(0.30)	-0.40	206
Jobless Claims	-0.28	(0.08)	-0.44	-0.44	(0.06)	-0.58	-1.09	(1.02)	-2.53	887
Mortgage App.	-0.28	(0.07)	-0.72	-0.00	(0.07)	-0.37	-1.17	(1.24)	-5.81	570

Relaxing rational expectations: private information

Tbl 6: Omitting restrictions that recovered probabilities are rational leaves point estimates unchanged, but standard errors are larger

Panel A: One-time Signal

	$RRA = 10,\ EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			
Event	Ω	$se(\Omega)$	$p\left(\chi^2\right)$	Ω	$se(\Omega)$	$p\left(\chi^2\right)$	Ω	$se(\Omega)$	$p\left(\chi^2\right)$	Obs
Consumer Comf.	0.039	(0.003)	0.351	0.035	(0.004)	0.998	0.405	(0.042)	0.350	574
Employment	0.053	(0.005)	1.000	0.059	(0.007)	1.000	0.540	(0.053)	1.000	207
FOMC	0.039	(0.006)	1.000	0.042	(0.009)	1.000	0.355	(0.053)	1.000	133
Pre-FOMC	0.042	(0.009)	1.000	0.034	(0.006)	1.000	0.466	(0.107)	1.000	134
GDP	0.034	(0.003)	1.000	0.033	(0.004)	1.000	0.349	(0.034)	1.000	206
Jobless Claims	0.043	(0.003)	0.001	0.042	(0.003)	0.318	0.441	(0.031)	0.001	887
Mortgage App.	0.037	(0.003)	0.449	0.035	(0.004)	0.994	0.368	(0.032)	0.443	570

Panel B: Signal Every Period

	$RRA = 10,\ EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			
Event	Ω	$se(\Omega)$	$p\left(\chi^2\right)$	Ω	$se(\Omega)$	$p\left(\chi^2\right)$	Ω	$se(\Omega)$	$p\left(\chi^2\right)$	Obs
Consumer Comf.	13.95	(1.24)	0.35	8.27	(0.77)	0.99	77.56	(3.30)	0.35	574
Employment	22.93	(2.61)	1.00	12.13	(1.08)	1.00	85.82	(2.49)	1.00	207
FOMC	16.21	(2.65)	1.00	8.68	(1.70)	1.00	74.36	(4.99)	1.00	133
Pre-FOMC	17.14	(3.35)	1.00	7.42	(1.39)	1.00	79.41	(6.99)	1.00	134
GDP	14.29	(1.69)	1.00	7.10	(0.69)	1.00	70.88	(3.49)	1.00	206
Jobless Claims	17.89	(1.20)	0.00	8.92	(0.53)	0.32	78.90	(2.18)	0.00	887
Mortgage App.	13.09	(0.97)	0.45	8.27	(0.77)	0.99	74.36	(2.70)	0.44	570

Relaxing rational expectations: private information

Tbl 6: Omitting restrictions that recovered probabilities are rational leaves point estimates unchanged, but standard errors are larger

Panel A: One-time Signal

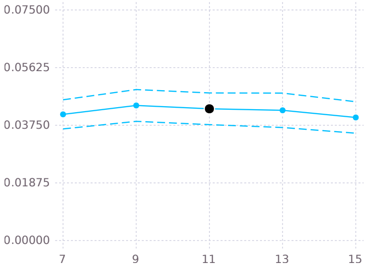
	$RRA = 10,\ EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			
Event	Ω	$se(\Omega)$	$p\left(\chi^2\right)$	Ω	$se(\Omega)$	$p\left(\chi^2\right)$	Ω	$se(\Omega)$	$p\left(\chi^2\right)$	Obs
Consumer Comf.	0.039	(0.003)	0.351	0.035	(0.004)	0.998	0.405	(0.042)	0.350	574
Employment	0.053	(0.005)	1.000	0.059	(0.007)	1.000	0.540	(0.053)	1.000	207
FOMC	0.039	(0.006)	1.000	0.042	(0.009)	1.000	0.355	(0.053)	1.000	133
Pre-FOMC	0.042	(0.009)	1.000	0.034	(0.006)	1.000	0.466	(0.107)	1.000	134
GDP	0.034	(0.003)	1.000	0.033	(0.004)	1.000	0.349	(0.034)	1.000	206
Jobless Claims	0.043	(0.003)	0.001	0.042	(0.003)	0.318	0.441	(0.031)	0.001	887
Mortgage App.	0.037	(0.003)	0.449	0.035	(0.004)	0.994	0.368	(0.032)	0.443	570

Panel B: Signal Every Period

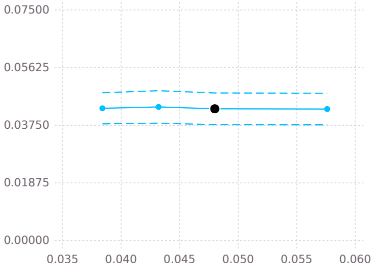
	$RRA = 10,\ EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			
Event	Ω	$se(\Omega)$	$p\left(\chi^2\right)$	Ω	$se(\Omega)$	$p\left(\chi^2\right)$	Ω	$se(\Omega)$	$p\left(\chi^2\right)$	Obs
Consumer Comf.	13.95	(1.24)	0.35	8.27	(0.77)	0.99	77.56	(3.30)	0.35	574
Employment	22.93	(2.61)	1.00	12.13	(1.08)	1.00	85.82	(2.49)	1.00	207
FOMC	16.21	(2.65)	1.00	8.68	(1.70)	1.00	74.36	(4.99)	1.00	133
Pre-FOMC	17.14	(3.35)	1.00	7.42	(1.39)	1.00	79.41	(6.99)	1.00	134
GDP	14.29	(1.69)	1.00	7.10	(0.69)	1.00	70.88	(3.49)	1.00	206
Jobless Claims	17.89	(1.20)	0.00	8.92	(0.53)	0.32	78.90	(2.18)	0.00	887
Mortgage App.	13.09	(0.97)	0.45	8.27	(0.77)	0.99	74.36	(2.70)	0.44	570

Modifying the empirical design I

Fig 5: One-time signal of jobless claims



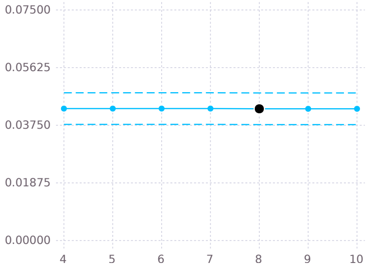
(a) Number of states, n



(b) State spacing, dk

Modifying the empirical design II

Fig 5: One-time signal of jobless claims



(a) Optimization precision

Relation to entropy and the role of rational expectations

- For intuition, consider the simpler case of log utility:

$$\omega(z_t;\alpha) = \underbrace{\beta\sum_{s_t}\alpha(s_t|z_t)\left[H(z_t;\alpha_0) - H(z_t,s_t;\alpha)\right]}_{\text{Expected reduction in entropy (uncertainty of p)}} + \underbrace{\beta\sum_{z_{t+1}}\omega(z_{t+1};\alpha)\,p(z_{t+1}|z_t)}_{\text{Present value of future signals}}$$

$$+\beta\underbrace{\sum_{z_{t+1}}\left[\sum_{s_t}\alpha(s_t|z_t)\,p(z_{t+1}|z_t,s_t) - p(z_{t+1}|z_t)\right] r(z_{t+1}|z_t)}_{\text{Belief errors covariance with log returns}}$$

$$+\beta\underbrace{\sum_{z_{t+1}}\left[\sum_{s_t}\alpha(s_t|z_t)\,p(z_{t+1}|z_t,s_t) - p(z_{t+1}|z_t)\right] v(z_{t+1};\alpha_0)}_{\text{Belief errors covariance with continuation value/cosumption ratios}}$$

- If one assumes (we do not) rational expectations (law of total probability) then last two terms drop out
  - Turns out to be empirically important

Relation to entropy and the role of rational expectations

- For intuition, consider the simpler case of log utility:

$$\omega\left(z_t;\alpha\right)=\underbrace{\beta\sum_{s_t}\alpha\left(s_t|z_t\right)\left[H\left(z_t;\alpha_0\right)-H\left(z_t,s_t;\alpha\right)\right]}_{\text{Expected reduction in entropy (uncertainty of p)}}+\underbrace{\beta\sum_{z_{t+1}}\omega\left(z_{t+1};\alpha\right)p\left(z_{t+1}|z_t\right)}_{\text{Present value of future signals}}$$

$$+\underbrace{\beta\sum_{z_{t+1}}\left[\sum_{s_t}\alpha\left(s_t|z_t\right)p\left(z_{t+1}|z_t,s_t\right)-p\left(z_{t+1}|z_t\right)\right]r\left(z_{t+1}|z_t\right)}_{\text{Belief errors covariance with log returns}}$$

$$+\underbrace{\beta\sum_{z_{t+1}}\left[\sum_{s_t}\alpha\left(s_t|z_t\right)p\left(z_{t+1}|z_t,s_t\right)-p\left(z_{t+1}|z_t\right)\right]v\left(z_{t+1};\alpha_0\right)}_{\text{Belief errors covariance with continuation value/cosumption ratios}}$$

- If one assumes (we do not) rational expectations (law of total probability) then last two terms drop out
  - Turns out to be empirically important

Relation to entropy and the role of rational expectations

- For intuition, consider the simpler case of log utility:

$$\omega\left(z_t;\alpha\right) = \underbrace{\beta \sum_{s_t} \alpha\left(s_t|z_t\right)\left[H\left(z_t;\alpha_0\right) - H\left(z_t,s_t;\alpha\right)\right]}_{\text{Expected reduction in entropy (uncertainty of p)}} + \underbrace{\beta \sum_{z_{t+1}} \omega\left(z_{t+1};\alpha\right) p\left(z_{t+1}|z_t\right)}_{\text{Present value of future signals}}$$

$$+ \underbrace{\beta \sum_{z_{t+1}} \left[\sum_{s_t} \alpha\left(s_t|z_t\right) p\left(z_{t+1}|z_t,s_t\right) - p\left(z_{t+1}|z_t\right)\right] r\left(z_{t+1}|z_t\right)}_{\text{Belief errors covariance with log returns}}$$

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The role of rational expectations

Tbl 4: Ordering changes considerably once we allow for deviations from rational expectations (law of total probability for option-implied distributions)

Panel A: One-time Signal

Event	$\Omega = 1 - e^{-\omega}$	$\omega = \omega^p + \omega^q + \omega^v$	ω^p	ω^q	ω^v	Obs
Consumer Comf.	0.417	0.417	-0.070	0.487	0.000	574
Employment	0.565	0.567	0.479	0.088	-0.000	207
FOMC	0.341	0.342	0.211	0.131	-0.000	133
Pre-FOMC	0.406	0.407	0.233	0.173	0.000	134
GDP	0.364	0.365	0.124	0.241	-0.000	206
Jobless Claims	0.445	0.446	-0.062	0.509	0.000	887
Mortgage App.	0.357	0.358	0.057	0.301	0.000	570

Panel B: Signal Every Period

Event	$\Omega = 1 - e^{-\omega}$	$\omega = \omega^p + \omega^q + \omega^v$	ω^p	ω^q	ω^v	Obs
Consumer Comf.	78.19	152.28	-36.47	188.77	-0.02	574
Employment	86.47	200.04	172.32	27.69	0.02	207
FOMC	69.76	119.59	82.19	37.46	-0.06	133
Pre-FOMC	75.43	140.35	105.83	34.47	0.06	134
GDP	71.55	125.70	36.20	89.52	-0.02	206
Jobless Claims	78.70	154.64	-39.83	194.51	-0.04	887
Mortgage App.	72.69	129.78	36.52	93.26	-0.00	570

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Conclusion

- We derive an expression for the value of information to an investor in a dynamic environment with recursive utility
- We estimate the value of key macroeconomic indicators from changes in index option prices
  - One-time signal vs. signal every period
  - Psychic vs. instrumental values
  - Private vs. public information
- Comparative statics are rather intuitive
- Future research may use our methodology to study the value of information at the firm level (M&A, earnings, etc.)

Estimation: employment example

- Employment reported on the first Friday of each month
- We estimate state prices at market close of preceding Thursday (date t)
- We estimate physical probabilities on preceding Thursday (date t) and at market close of release Friday (date $t + dt$)
- We consider the information structure just before the information release as α_0 and the one just after the information release as α
- Applying GMM we estimate the value of information $\omega_i(\alpha)$ for each state $i = 1, ..., n$.

Estimation: state space and state prices

- Following Ross (2015), on each date t we discretize the state relative to the current spot price of SPX into 11 possible equally spaced log-returns in $[-0.24, 0.24]$
- We focus on a one-month horizon
- Thought exercise: How much would you be willing to pay for obtaining an information source early on a monthly basis?
- A state price $q\left(z'|z\right)$ corresponds to the price of a security paying \$1 if state z' is realized in one month given that the current state is z
- We calculate state prices from S&P 500 options using the Breeden & Litzenberger (1978) method by estimating the implied volatility surface using the Carr and Wu (2010) method

Event summary statistics

Ordering changes considerably once we allow for deviations from rational expectations (law of total probability for option-implied distributions)

	Levels on event day				Changes from previous day				
Event	$E[r^e]$	$\sigma[r^e]$	SR	H^p	$\Delta E[r^e]$	$\Delta\sigma[r^e]$	ΔSR	ΔH^p	Obs
All	5.93	19.89	26.69	149.50	0.00	-0.00	0.02	0.00	4931
	(0.07)	(0.09)	(0.16)	(0.41)	(0.02)	(0.02)	(0.05)	(0.08)	
Consumer Comf.	5.43	18.34	25.36	140.13	-0.06	-0.07	-0.39	-0.25	606
	(0.24)	(0.27)	(0.51)	(1.24)	(0.05)	(0.05)	(0.13)	(0.23)	
Employment	5.96	19.83	26.61	149.00	-0.30	-0.25	-1.16	-1.19	219
	(0.34)	(0.43)	(0.78)	(2.04)	(0.08)	(0.09)	(0.26)	(0.41)	
FOMC	6.11	19.94	27.47	149.63	-0.33	-0.60	-0.61	-2.76	147
	(0.44)	(0.50)	(0.97)	(2.37)	(0.13)	(0.10)	(0.30)	(0.44)	
Pre-FOMC	6.44	20.55	28.06	152.38	0.19	0.08	0.82	0.21	147
	(0.45)	(0.52)	(1.00)	(2.40)	(0.16)	(0.10)	(0.36)	(0.43)	
GDP	5.84	19.97	26.20	149.75	-0.11	-0.13	-0.29	-0.61	222
	(0.33)	(0.41)	(0.73)	(1.93)	(0.08)	(0.07)	(0.22)	(0.33)	
Jobless Claims	6.14	19.94	27.48	149.50	0.00	-0.05	-0.02	-0.18	948
	(0.17)	(0.20)	(0.38)	(0.95)	(0.04)	(0.04)	(0.11)	(0.18)	
Mortgage App.	5.51	18.31	25.86	139.75	0.07	-0.15	0.45	-0.89	601
	(0.24)	(0.28)	(0.51)	(1.27)	(0.05)	(0.05)	(0.13)	(0.22)	
Historical	6.00	19.49	30.81						4931
	(0.28)	(0.20)	(22.61)						

Relation to entropy

- Cabrales, Gossner, and Serrano (2013 AER) focus on a log utility agent, faced with a static investment problem
- The value of information in that case equals the mean reduction in entropy that the information source can generate

$$I^p(z;\alpha) \equiv \sum_s \left[H(z;\alpha_0) - H(z,s;\alpha)\right]\alpha(s|z)$$

Relation to entropy

- Denote

$$H\left(z,s;\alpha\right)\equiv-\sum_{z'}p\left(z'|z,s\right)\log p\left(z'|z,s\right)$$

  the entropy of the future state z' distribution given the current state z and signal s
- Similarly, $H\left(z;\alpha_0\right)\equiv-\sum_{z'}p\left(z'|z\right)\log p\left(z'|z\right)$ is the unconditional entropy in state z
- Entropy is a measure of the dispersion of the probability distribution
- $H\left(z,\alpha_0\right)-H\left(z,s;\alpha\right)$, the reduction in entropy associated with signal s, is a measure of the information in this signal