Fundamentals of Perpetual Futures

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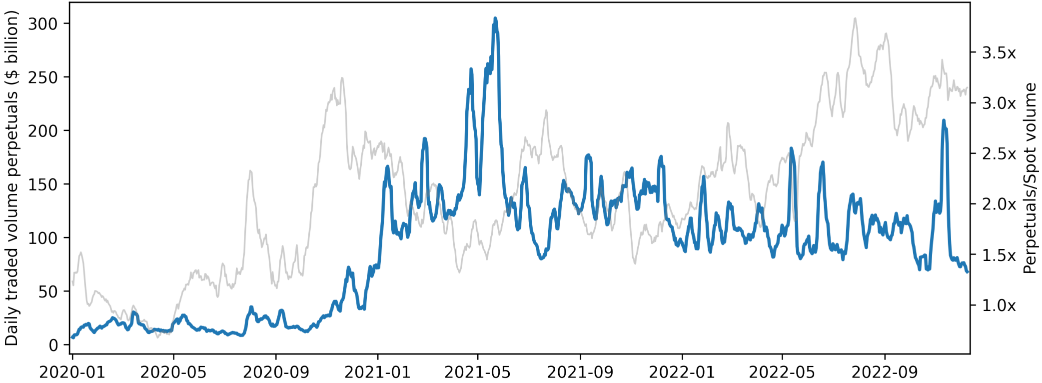
What are perpetual futures?

- Perpetual futures are swaps that never expire
- Efficient way to hedge and speculate
  - Allow high leverage
  - No need to take delivery of crypto
  - No rollover
  - Concentrates liquidity
- First introduced in Shiller (1993)

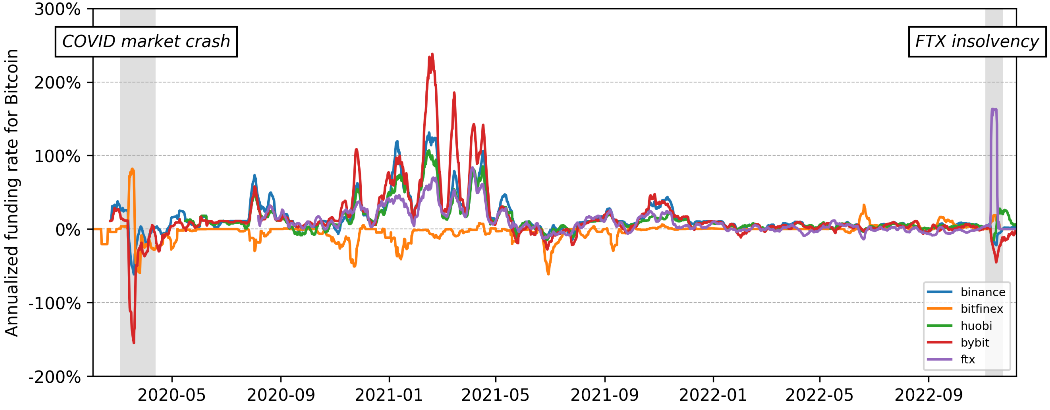
Most popular derivative traded in crypto markets

$100 billion daily volume



Among the most actively-traded products on FTX before it imploded

Alameda Research was counterparty to many such leveraged trades



Questions

1. What are the fundamental values of perpetual futures in theory?
2. How large are deviations from these fundamentals in practice?

Our paper

Theoretic contribution

- Derive no-arbitrage prices for perpetual futures in frictionless markets
- Derive no-arbitrage bounds for markets with trading costs

Empirical findings

- Deviations of crypto perpetual futures from no-arbitrage prices are large
- Deviations comove across cryptocurrencies
- Diminish over time as crypto markets develop and become more efficient
- Simple trading strategy generates large Sharpe ratios

Related work

- Descriptive evidence of a "carry trade" in perpetual futures: Alexander, Choi, Park, and Sohn (2020), Franz and Schmeling (2021), De Blasis and Webb (2022), Ferko, Moin, Onur, and Penick (2022), Christin, Routledge, Soska, and Zetlin-Jones (2023)
- Theory on perpetuals: Angeris, Chitra, Evans, and Lorig (2022) derive no-arb prices for perpetuals assuming payoff is a fixed function of the spot price
- Recent work on fixed-maturity crypto futures by Schmeling, Schrimpf, and Todorov (2022) find large profits to carry strategies (Du, Tepper, and Verdelhan, 2018); Koijen, Moskowitz, Pedersen, and Vrugt, 2018), Cong, He, and Tang (2022)

Fixed-maturity forward refresher

Definition

- Two counterparties agree at time $t = 0$ to exchange underlying asset at future expiration time τ
- Underlying asset price S_t fluctuates over time
- Forward price $F_{0,\tau}$ is fixed at initiation
- At expiration, short counterparty delivers to long counterparty the underlying in exchange for the forward price

Fixed-maturity forward refresher

No arbitrage pricing

- In the absence of arbitrage, the forward price $F_{0,\tau}$ only depends on the initial underlying price S_0 and the interest rate r

$$F_{0,\tau} = S_0 e^{r\tau} \tag{1}$$

- Just the forward value of the spot price
- Easily extends to dividend-paying assets and storage costs

Fixed-maturity forward refresher

Arbitrage strategy

- Suppose the forward price is actually higher

$$F_{0,\tau} > S_0 e^{r\tau}$$

- Arbitrageur would today
  1. Open a short forward position (0 cashflow today)
  2. Borrow dollars in cash markets $(+S_0)$
  3. Buy the underlying at spot price $(-S_0)$
- At expiration time τ
  1. Collect forward price $(+F_{0,\tau})$
  2. Deliver underlying asset
  3. Repay dollar loan $(-S_0 e^{r\tau})$
- Net cashflow at expiration $F_{0,\tau} - S_0 e^{r\tau} > 0$

Perpetual futures (swap)

Definition

- No expiration date!
- No initial cash
- Can be closed at any time
- To stay in the contract, the long and short sides have to meet two requirements
- (1) Exchange at each interval ds a funding payment:

$$\text{Funding payment}_s = \kappa(F_s - S_s)ds.$$

In most exchanges, funding payments are paid every 8 hours and are proportional to the futures-spot spread. In this case, $\kappa = 3 \times 365 = 1095$.

- (2) Post mark-to-market margins to cover any losses.
- Arbitrage is risky: nothing guarantees that futures price converge to the spot!

Random-maturity arbitrage

Certain profits at an uncertain future time

Definition (Traditional)

A riskless arbitrage opportunity is defined with respect to payoff x at a certain future time T and its price $p(x)$. If (1) $x \geq 0$ almost surely, (2) $x > 0$ with some positive probability, (3) its price satisfies $p(x) \leq 0$, then this payoff is an arbitrage opportunity.

Definition (Our extension)

A random-maturity arbitrage opportunity is defined with respect to a bounded random payoff x at a bounded future random time $\tau < \bar{T}$, and its price $p(x)$. If (1) $x \geq 0$ almost surely, (2) $x > 0$ with some positive probability, (3) its price satisfies $p(x) \leq 0$, then this payoff is a random-maturity arbitrage opportunity.

Random-maturity arbitrage

Objection

- “But random-maturity arbitrage opportunities are not riskless!”
- Yes, but no-arbitrage prices are always just a useful fiction
- Real markets have transactions costs, margin requirements, and risk of liquidation
- Random-maturity no arb prices are similarly a useful benchmark

Perpetual futures - No arbitrage pricing

Assumptions

A1 *The risk-free rate in cash market r and the instantaneous interest rate on the underlying asset r' are constant.*

A2 *The gap between the perpetual futures and its fundamental price is bounded:* $\sup_t |F_t(\omega) - \lambda S_t(\omega)| < M, \forall \omega$, *where λ is the proportion constant in equation 2.*

A3 *The funding rate is larger than the risk-free rate in the cash market: $\kappa > r$.*

Proposition

Random-maturity arbitrage opportunities are absent if and only if

$$F_t = \frac{\kappa}{\kappa - (r - r')} S_t \tag{2}$$

The perpetual futures to spot gap is small when the interest rate spread $r - r'$ is low relative to the funding rate coefficient κ.

Perpetual futures

Arbitrage strategy

- Suppose the futures price is actually higher

$$F_0 > \frac{\kappa}{\kappa - (r - r')} S_0 = \lambda S_0.$$

- Arbitrageur would today
  1. Open a short futures position (0 cashflow today)
  2. Borrow dollars in cash markets $(+\lambda S_0)$
  3. Buy λ underlying at spot price $(-\lambda S_0)$
- At (random) unwinding time t
  1. Close futures position $(-\int_0^t e^{-rs} dF_s)$
  2. Proceeds from the spot market $(\kappa \int_0^t e^{-rs}(dS_s + r' S_s ds))$
  3. Interest payment of dollar loan $(-\lambda \int_0^t e^{-rs} r S_s ds)$
  4. Funding payments accrued $(\kappa \int_0^t (F_s - S_s) e^{-rs} ds)$
- Net discounted payoff $= -\int_0^t d(F_s - \lambda S_s) + \kappa \int_0^t e^{-rs}(F_s - \lambda S_s) ds$

Perpetual futures

Arbitrage strategy (continued)

- Suppose this is not an arbitrage opportunity
- Denote the deviation by: $u_s \equiv F_s - \lambda S_s$
- Net discounted payoff $= -\int_0^t du_s + \kappa \int_0^t e^{-rs} u_s ds = -e^{-rt} u_t + u_0 + (\kappa - r) \int_0^t u_s e^{-rs} ds$
- Then for all $t < \overline{T}$, the payoff is nonpositive

$$\underbrace{u_0}_{\text{traditional spread}} + \underbrace{(\kappa - r) \int_0^t u_s e^{-rs} ds}_{\text{funding payments}} \leq \underbrace{e^{-rt} u_t}_{\text{spread at unwinding}} \qquad (3)$$

- Turns out we can bound this process from below and show that:

$$e^{\kappa t} u_0 \leq u_t \qquad (4)$$

- But $u_0 > 0$ implies LHS $\to \infty$ at speed of $e^{\kappa t}$, which violates ounded spread assumption

$\Rightarrow$ Arbitrage! Some t exists when arb can unwind at a positive discounted payoff

Perpetual futures - No arbitrage bounds

Proposition

With constant round-trip trading costs $C > 0$, the absence of random-maturity arbitrage opportunities implies that the perpetual futures price must lie within the following bound relative to the spot:

$$\left| F_t - \frac{\kappa}{\kappa - (r - r')} S_t \right| \leq C \quad (5)$$

Data

Focus on the 5 largest cryptocurrencies with a total market cap of $1.83 trillion
73.2% of the Crypto market in March 2024

Crypto	start date	end date	N
BTC	2020-01-08	2024-03-11	36,578
ETH	2020-01-08	2024-03-11	36,578
BNB	2020-02-10	2024-03-11	35,777
DOGE	2020-07-10	2024-03-11	32,152
ADA	2020-01-31	2024-03-11	36,017

- Perpetual futures, spot, and funding rate at 1-hour frequency from Binance
- Funding rate is paid every 8 hours
- Market is open 24/7
- Risk-free interest rates from Aave

Deviations from no-arbitrage benchmarks

Define ρ as the annualized interest rate deviation that rationalizes an observed future-spot spread:

$$F = \frac{\kappa}{\kappa - (r + \rho - r')} S.$$

or approximately

$$\rho \approx \kappa \log(F/S) - (r - r'),$$

Deviations of perpetual futures from no-arbitrage benchmarks

Mean deviation is no different than zero

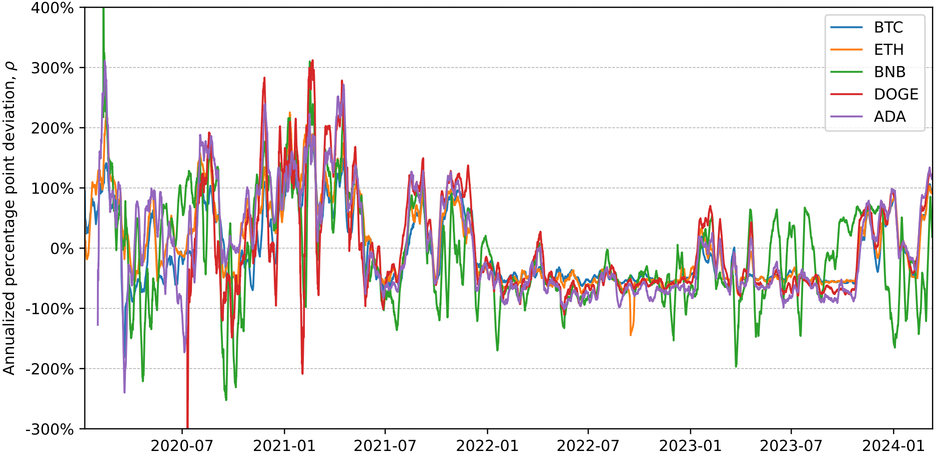
Mean **absolute** deviation is about 60–90% per year

Considerably larger than deviations Du-Tepper-Verdelhan (2018) find in traditional FX markets

	ρ				$\lvert\rho\rvert$			
Asset	Mean	Median	Std	p-value	Mean	Median	Std	p-value
BTC	-0.06	-0.32	0.74	0.80	0.57	0.51	0.48	0.00
ETH	0.04	-0.24	0.85	0.31	0.63	0.53	0.57	0.00
BNB	-0.10	-0.17	1.22	0.85	0.88	0.70	0.85	0.00
DOGE	0.01	-0.28	1.38	0.45	0.86	0.64	1.07	0.00
ADA	0.03	-0.21	1.14	0.39	0.83	0.70	0.79	0.00

Deviations of perpetual futures from no-arbitrage benchmarks

Strong comovement
Deviations shrink in mid 2021



Random-maturity arbitrage strategy

No arbitrage bounds prescribe a threshold strategy to exploit divergence from fundamentals

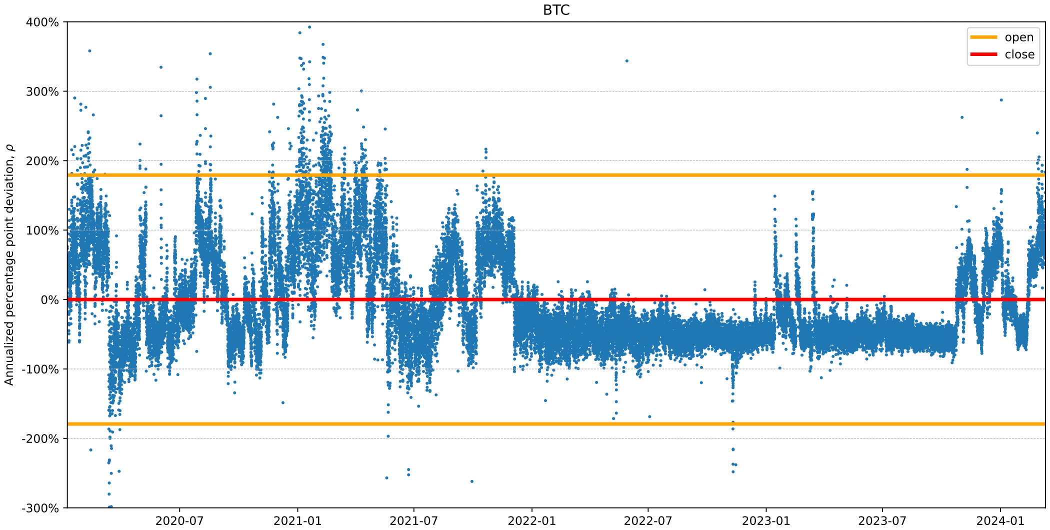
Fee levels	Spot	Futures	ρ_l	ρ_u
No	0%	0%	0.0%	0.0%
Low	0.0225%	0.0018%	-53.2%	53.2%
Medium	0.045%	0.0072%	-114.4%	114.3%
High	0.0675%	0.0144%	-179.5%	179.2%

Given C = round-trip percentage trading costs

- if $\rho > \rho_u$
- then borrow cash, buy λ spot and short the future
- collect funding rate and pay interest
- close the positive when ρ is back to zero

Opposite strategy if $\rho < \rho_l$

Random-maturity arbitrage strategy: BTC, high trading costs



Performance of Random-maturity arbitrage strategy

		Fee tiers			
		No	Low	Medium	High
BTC	SR	3.53	2.20	2.16	1.80
	Return	13.70	8.40	7.93	6.38
	Volatility	3.88	3.82	3.68	3.55
	MaxDD	-4.24	-4.42	-4.34	-4.43
	α	13.36	6.46	5.94	4.38
	t_α	5.34	2.76	2.58	1.89
	Active %	100.00	85.28	36.03	20.06
	OtC time	19.22	86.73	106.21	134.94
ETH	SR	4.83	2.95	2.86	2.55
	Return	21.23	12.71	11.61	9.59
	Volatility	4.40	4.30	4.06	3.76
	MaxDD	-4.13	-4.21	-3.91	-3.94
	α	26.00	13.62	11.80	9.49
	t_α	7.26	4.31	4.06	3.54
	Active %	99.94	83.93	40.35	22.68
	OtC time	14.57	52.60	82.95	83.71
BNB	SR	12.31	7.40	5.76	4.84
		⋮			
DOGE	SR	10.35	6.69	4.85	3.58
		⋮			
ADA	SR	10.43	5.53	3.46	2.68
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Performance over time: Unrestricted, High trading costs tier

		2020	2021	2022	2023	2024	All
BTC	SR	2.26	2.39	0.70	1.32	11.52	1.80
	Return	8.29	14.81	0.28	1.11	11.97	6.38
	Volatility	3.67	6.18	0.40	0.84	1.04	3.55
	MaxDD	-1.90	-4.43	-0.29	-0.24	-0.15	-4.43
	Active %	28.66	34.43	9.21	7.68	22.18	20.06
	OtC time	94.00	141.52	402.50	223.33	185.50	134.94
	N	8,616	8,760	8,760	8,760	1,682	36,578
ETH	SR	3.08	3.52	1.29	1.64	10.70	2.55
	Return	17.12	18.08	1.19	1.81	10.98	9.59
	Volatility	5.55	5.13	0.92	1.10	1.03	3.76
	MaxDD	-3.94	-2.49	-0.45	-0.30	-0.16	-3.94
	Active %	37.80	34.91	8.23	10.35	20.93	22.68
	OtC time	78.54	67.81	102.14	180.40	175.00	83.71
	N	8,616	8,760	8,760	8,760	1,682	36,578
BNB	SR	6.04	6.60	2.83	2.96	4.31	4.84
			⋮				
DOGE	SR	4.90	5.93	1.49	0.85	2.75	3.58
			⋮				
ADA	SR	3.88	3.41	2.33	1.12	2.23	2.68
			⋮				

Performance over time: Long-spot only, High trading costs tier

		2020	2021	2022	2023	2024	All
BTC	SR	1.98	2.28	0.51	0.87	11.52	1.62
	Return	6.43	13.95	0.08	0.26	11.97	5.49
	Volatility	3.25	6.13	0.17	0.30	1.04	3.40
	MaxDD	-1.90	-4.43	-0.04	-0.24	-0.15	-4.43
	Active %	22.28	32.15	0.02	2.85	22.18	14.66
	OtC time	111.94	198.36	1.00	124.00	185.50	147.92
	N	8,616	8,760	8,760	8,760	1,682	36,578
ETH	SR	2.57	3.36	0.81	1.16	10.70	2.23
	Return	13.98	17.14	0.53	0.48	10.98	8.15
	Volatility	5.44	5.09	0.66	0.42	1.03	3.66
	MaxDD	-3.94	-2.49	-0.26	-0.30	-0.16	-3.94
	Active %	34.48	34.59	0.08	3.76	20.93	18.29
	OtC time	122.79	74.10	1.33	108.67	175.00	93.21
	N	8,616	8,760	8,760	8,760	1,682	36,578
BNB	SR	4.46	4.99	0.76	0.73	2.24	3.29
			⋮				
DOGE	SR	2.64	5.03	1.03	0.49	2.75	2.52
			⋮				
ADA	SR	3.16	2.94	-	0.88	2.23	2.11
			⋮				

Returns mostly come from price convergence rather than funding payments

		2020	2021	2022	2023	2024	All
BTC	Return	21.68	29.60	0.35	2.65	17.21	13.70
	Price	14.00	14.29	2.29	3.59	11.19	8.64
	Funding	7.68	15.31	-1.94	-0.94	6.03	5.06
ETH	Return	40.93	36.66	3.15	5.52	15.95	21.23
	Price	27.58	19.07	3.68	5.75	8.97	13.73
	Funding	13.35	17.59	-0.53	-0.24	6.97	7.49
BNB	Return	82.48	76.29	27.75	39.14	36.92	54.81
	Price	66.70	60.37	21.69	32.26	21.68	43.58
	Funding	15.78	15.92	6.06	6.89	15.24	11.23
DOGE	Return	220.81	113.79	42.59	8.76	25.98	75.13
	Price	214.72	93.24	43.59	8.51	15.74	68.40
	Funding	6.09	20.55	-1.00	0.25	10.24	6.72
ADA	Return	90.81	66.73	43.48	23.56	42.55	54.83
	Price	77.17	49.51	42.67	23.71	32.92	46.98
	Funding	13.64	17.22	0.81	-0.15	9.64	7.85

Explaining deviations

Regression of the futures-spot gap against explanatory variables

Dependent Variable:	ρ			$\lvert\rho\rvert$		
Time	-0.23***	-0.14***	-0.25***	-0.11***	-0.08***	-0.10***
	(-4.52)	(-3.53)	(-6.47)	(-5.22)	(-4.88)	(-5.00)
Ret		0.13***	0.20***		0.05***	0.06***
		(4.52)	(10.43)		(4.07)	(5.00)
Vol			-0.04***			-0.01
			(-6.02)			(-1.50)
Currency FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.09	0.26	0.33	0.07	0.13	0.14
N	7235	7235	7235	7235	7235	7235

Takeaways

- We derive no-arbitrage prices and bounds for perpetual futures
- Provide valuable benchmarks + strategy to exploit deviations
- Find large deviations of crypto perpetual futures from no-arbitrage prices
- Deviations comove across cryptocurrencies and diminish over time
- Simple trading strategy generates large Sharpe ratios

AAVE Interest Rate